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Comparison of the Various Formulas used in The Design of Crane Hooks

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COMPARISON OF THE VARIOUS FORMULAS USED IN THE DESIGN OF CRANE HOOKS

BY

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THESIS FOR THE DEGREE OF BACHELOR OF SCIENCE

IN MECHANICAL ENGINEERING

IN THE

COLLEGE OF ENGINEERING

OF THE

UNIVERSITY OF ILLINOIS

PRESENTED JUNE, 1907

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HARRY PAUL HUMPHREYS and WALTER HENRY HUTH

ENTITLED COMPARISON OF THE VARIOUS FORMULAS

USED IN THE DESIGN OF CRANE HOOKS

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF Bachelor of Science in Mechanical Engineering

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Comparison of the Various Formulas Used in the Design of Crane Hooks.

Introduction

The crane hook is one of the important links in most all kinds of hoisting and conveying machinery of any size; and, as such, deserves the attention of the designer as much as other machine parts. However, it is a recognized fact that until very recently crane hooks have been very roughly designed, little attention being paid to the mathematical analysis of the stresses acting. It is true that the performances of hooks under actual load have given some very valuable hints as to proper cross sections and radius of curvature; but accurate results can only be obtained by formula.

However when we turn to the theory of stresses in crane hooks or curved bars generally, we find several theories which vary considerably in the methods of attacking the problem, but giving results

which agree fairly well. It is the purpose of this thesis to review four different theories and give numerical results of the stresses acting according to the several theories.

The theories to be reviewed are Unwin's theory of the stresses acting in eccentrically loaded straight bars, a theory which is most used in figuring crane hooks at present, but which is really inaccurate since the crane hook is a curved bar; Bach's theory of the stresses acting in curved bars; Slocum and Hancock's theory as given in their text book on the strength of materials; Andrew and Pearson's theory as given by Walter Rautenstrauch in the Journal of the Sibley College of Engineering of Cornell University.

Unwin's Theory

The theory of Unwin regarding the stresses in eccentrically loaded straight bars is as follows:-

In Fig. 1. is shown a straight prismatic bar with its gravity axis at AB, and acted upon by a force P, parallel to AB and at a distance p from it. The problem

then is to determine the distribution of stress in any right section as CD, and then find the maximum intensity of stress.

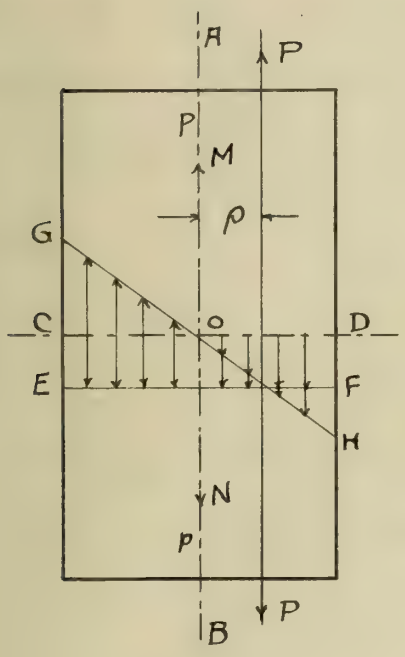


FIG. 1.

Without affecting the equilibrium of the system, we may introduce two equal and opposite forces of magnitude P along the gravity axis. The single force P has now been replaced by the centrally acting force OM, and the couple consisting of equal and opposite forces P and ON. The arm of this couple is ρ, and its magnitude Pρ.

The single force OM must be balanced by a stress in the section CD which is distributed uniformly over the section, and is equal to $\frac{P}{A}$, or

$$S_1 = \frac{P}{A}, \qquad (EC, Fig. 1.) \qquad (1)$$

where A is the area of the cross section.

The couple Pρ tends to rotate the bar counter-clockwise, and a stress of equal moment and opposite sense must balance this. The fibres to the right of AB will be in tension,

those to the left in compression; and according to Hooke's Law, the stress at any fibre is proportional to its distance from AB. Let DH represent the tensile stress S_2 at the outer fibre and CG be laid off to represent the compressive stress on the other side of AB. Also let c be the distance to the outer fibre.

Then $$\frac{S_x}{x} = \frac{S_2}{c}$$

or $$S_x = \frac{S_2 x}{c} \qquad (2)$$

Adding the stresses due to central loading loading and to the couple.

$$S_1 + S_x = S_1 + \frac{S_2 x}{c} \qquad (3)$$

According to the theory of flexure,

$$M = \frac{SI}{c},$$

Therefore, $$P_p = \frac{S_2 I}{c}$$

$$S_2 = \frac{P_p c}{I}$$

and $$S_1 + S_x = \frac{P}{A} + \frac{P_p x}{I} \qquad (4)$$

and $$S_1 + S_2 = \frac{P}{A} + \frac{P_p c}{I} \qquad (5)$$

It is evident from the formula, that maximum stresses may be found by simple substitution. The value of I, for irregular sections may be found graphically by Nehr's method. Fig 2. shows such

a graphical construction for one section.

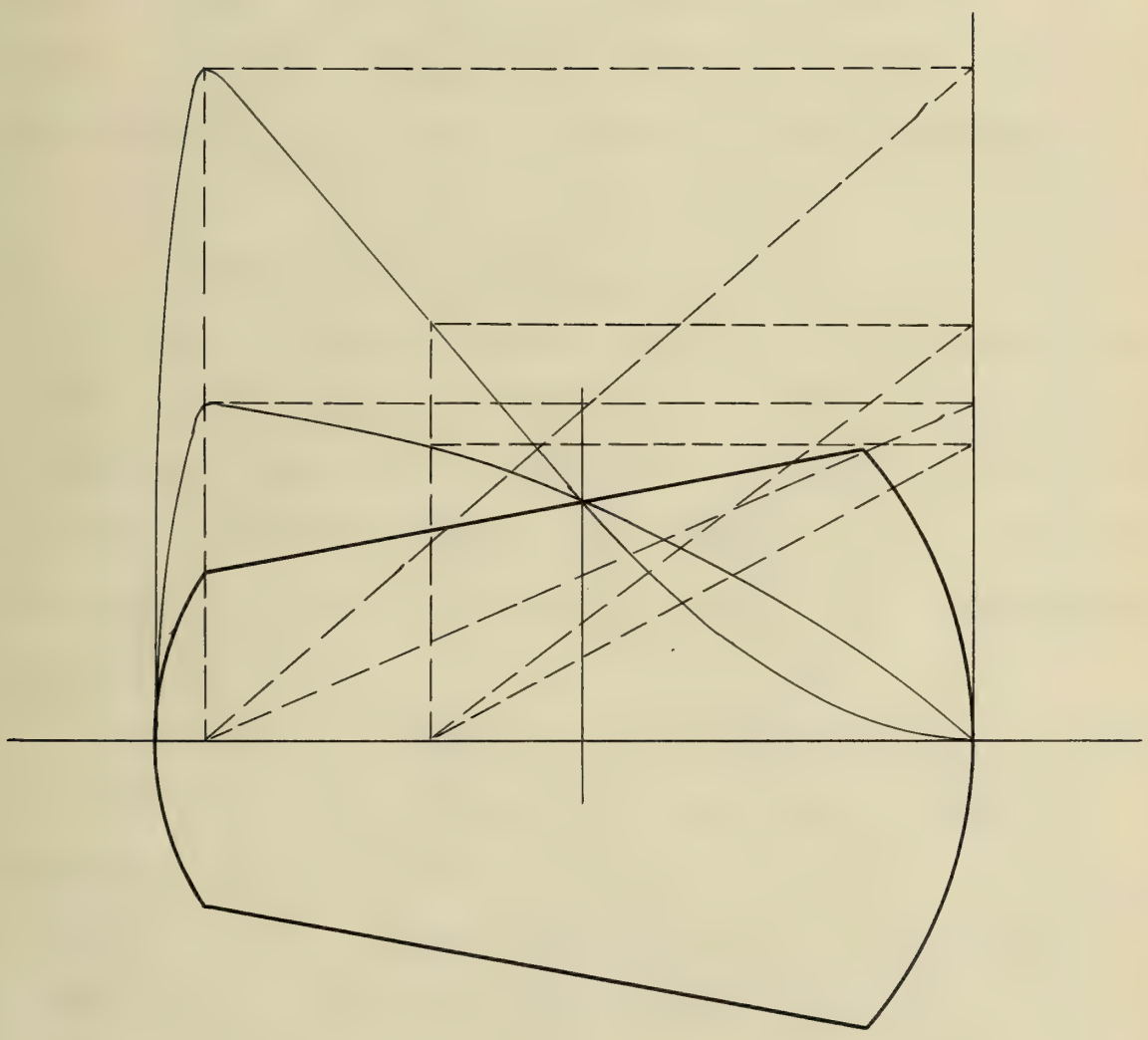


FIG. 2.

It is apparent that Unwin's theory does not take care, in the case of the curved bar, of the unequal elongations in the fibres at extrados and intrados, and the stresses acting as determined by this formula cannot be accurate, altho they have been in the past used in proportioning crane hook cross sections.

Bach's Theory

The other three theories investigated all take account of the curvature of the hook, and hence can be expected to give more nearly the actual stresses acting. According to Bach the stresses in curved bars may be analized as follows:-

Referring to Fig. 3, BCC_1B_1 represents a part of the body under consideration. OO_1 is the gravity axis of the section, and BOC and $B_1O_1C_1$ are two sections normal to this line. If OO_1 be a straight line, the sections BOC and $B_1O_1C_1$ are parallel; but should the center line be curved, the planes meet in the curvature axis at M, and make some angle $d\phi$ with each other.

Suppose a normal force P is uniformly distributed over the section $B_1O_1C_1$. If the sections were parallel, the extensions in all

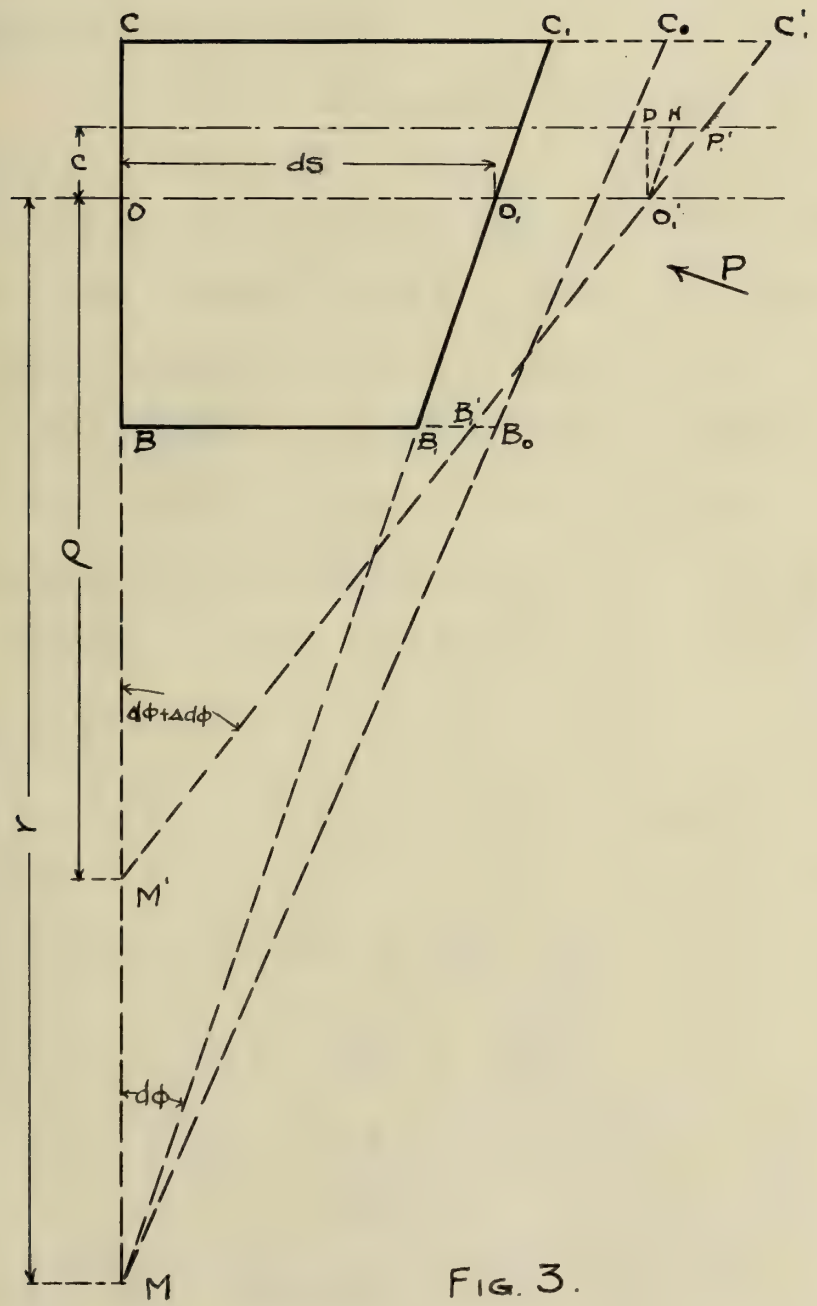


FIG. 3.

the fibres would be equal, and the parallelism of the sections would be undisturbed by the applied stress. When the sections make an angle $d\phi$ with each other, the extensions in the fibres are proportional to the length of the fibres, or to their distances from M.

Now suppose the section is acted upon by a normal force P and a couple whose moment may be denoted by M_b. The force P tends to make the section $B_1 O_1 C_1$ take the position $B_0 C_0$, while the couple of moment M_b brings the section to $B_1' O_1' C_1'$. The action of the couple shifts the center of curvature of OO_1, from M to M', and the radius of curvature is decreased from r to ρ. The angle between the sections is increased from $d\phi$ to $(d\phi + \Delta d\phi)$.

Let e_0 = the relative extension of the center line OO_1.

e = " " " " " fibre PP_1.

ds = " length of the fibre OO_1.

Then
$$e_0 = \frac{\Delta ds}{ds} = \frac{O_1 O_1'}{OO_1} \qquad (6)$$

$$e = \frac{P_1 P_1'}{PP_1} \qquad (7)$$

Now draw $O_1'H$ parallel to $B_1 C_1$,

Then
$$P_1 P_1' = P_1 H + H P_1' = O_1 O_1' + H P_1' \qquad (8)$$

$$\text{But} \quad O_1O_1' = e_0 (OO_1) = e_0 ds = e_0 r d\phi \tag{9}$$

From the geometry of the figure

$$HP_1' = O_1'H \times (d\phi + \Delta d\phi - d\phi) = c(\Delta d\phi)$$

$$PP_1 = (r+c)\, d\phi$$

Substituting these values in equation (7)

$$e = \frac{e_0 r d\phi + c\, \Delta d\phi}{(r+c)\, d\phi} = \frac{e_0 + \frac{c\, \Delta d\phi}{r d\phi}}{1 + \frac{c}{r}} \tag{10}$$

Let the ratio $\frac{\Delta d\phi}{d\phi}$ be denoted by w,

$$e = e_0 + (w - e_0)\, \frac{c}{r+c} \tag{11}$$

The normal stress corresponding to the extension e is

$$S = Ee = E\left[e_0 + (w - e_0)\, \frac{c}{r+c}\right] \tag{12}$$

in which E stands for the modulus of elasticity. Let A denote the area of the cross section, and dA an infinitely small element of area; placing the stresses in equilibrium with the external forces,

$$P = \int S\, dA = \int E\left[e_0 + (w - e_0)\, \frac{c}{r+c}\right] dA$$

$$M_b = \int cS\, dA = \int Ec\left[e_0 + (w - e_0)\, \frac{c}{r+c}\right] dA$$

Assuming now that E is constant,

$$P = E \int \left[e_0 + (w - e_0)\, \frac{c}{r+c}\right] dA \tag{13}$$

$$M_b = E \int c\left[e_0 + (w - e_0)\, \frac{c}{r+c}\right] dA \tag{14}$$

The center line OO_1 passes through the center of gravity of each normal section; therefore

$$\int c\, dA = 0$$

Now let $$\int \frac{c}{r+c}\, dA = -ZA \qquad (15)$$

Then $$\int \frac{c^2}{r+c}\, dA = \int \left(c - r\frac{c}{r+c}\right) dA = -r\int \frac{c}{r+c}\, dA = ZAr \qquad (16)$$

Substituting in (13) and (14)

$$P = EA\left[e_o - Z(w - e_o)\right] \qquad (17)$$

$$M_b = EA\left[w - e_o\right] rZ \qquad (18)$$

Solving for w and e_o, we get

$$w - e_o = \frac{M_b}{EArZ}$$

$$e_o = \frac{P}{EA} + Z\left[w - e_o\right] = \frac{1}{EA}\left[P + \frac{M_b}{r}\right]$$

$$w = e_o + \frac{1}{EA}\,\frac{M_b}{rZ} = \frac{1}{EA}\left[P + \frac{M_b}{r} + \frac{M_b}{rZ}\right]$$

Substituting in (12)

$$S = Ee = E\left[\frac{1}{EA}\left(P + \frac{M_b}{r}\right)\right] + E\,\frac{c}{r+c}\,\frac{M_b}{EAZr}$$

$$= \frac{P}{A} + \frac{M_b}{Ar} + \frac{c}{r+c}\,\frac{M_b}{ArZ}$$

$$S = \frac{P}{A} + \frac{M_b}{Ar}\left[1 + \frac{c}{r+c}\,\frac{1}{Z}\right] \qquad (19)$$

All the quantities in the above equation are directly obtainable from the hook dimensions, except Z, which, for irregular sections, must be found graphically. In Fig. 4. draw first a 45° line GB through the center of gravity G. Through any point of the section draw AC parallel to

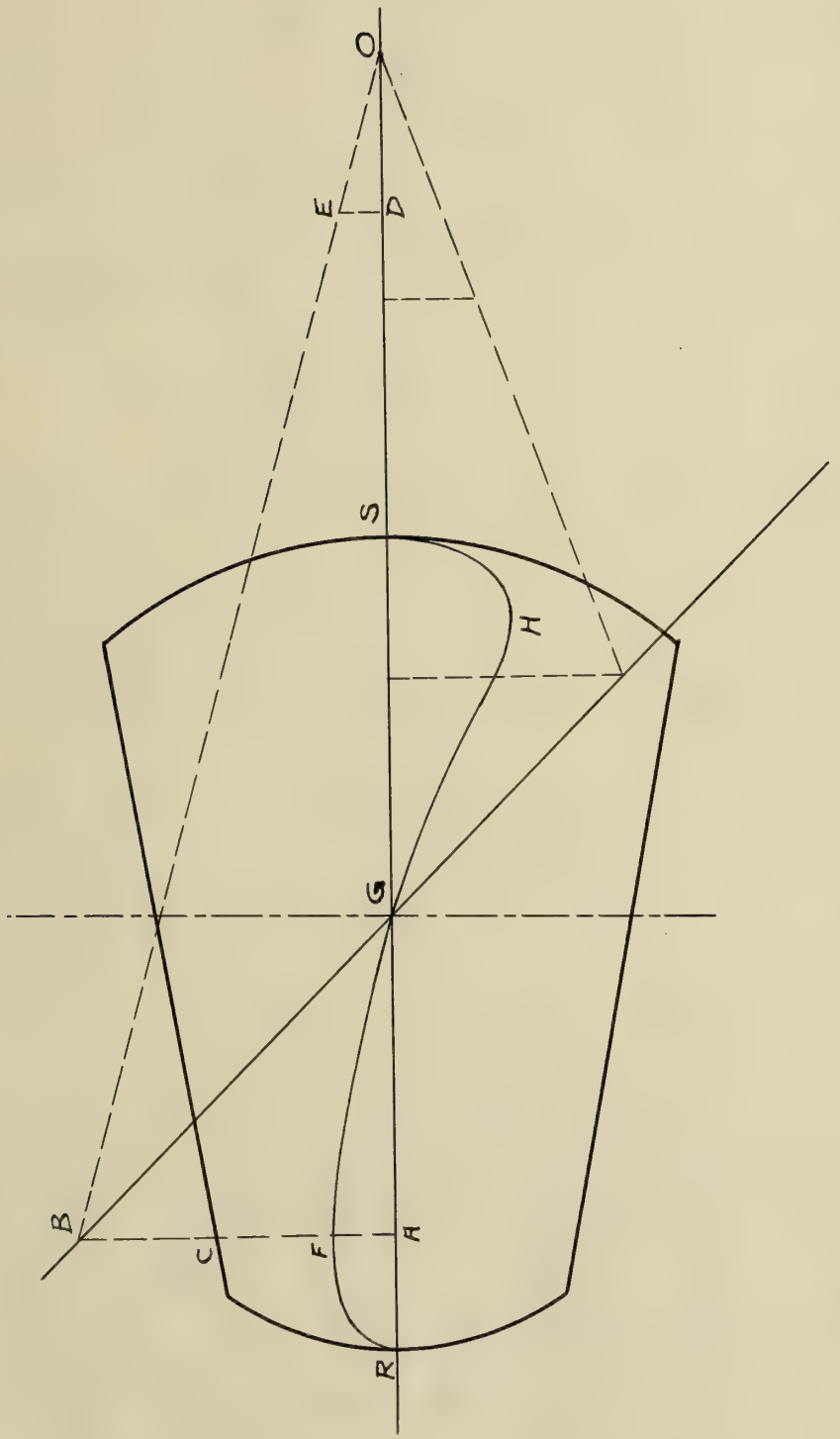


FIG. 4.

the gravity axis through G.

Then $AG = AB = c$ $\qquad OA = OG + GA = r + c$

Now lay off $OD = AC$; and draw DE parallel to AB. Then project E vertically to F.

From similar triangles

$$\frac{DE}{OD} = \frac{AB}{OA} \quad ; \quad \frac{AF}{AC} = \frac{AB}{OA} = \frac{c}{r+c}$$

Then $$AF = AC \frac{c}{r+c} \tag{20}$$

Let dc denote the width of an infinitesimal strip along AC.

Then $$AC \cdot dc = \tfrac{1}{2} dA$$

$$AF \cdot dc = AC \frac{c}{r+c} \cdot dc = \tfrac{1}{2} \frac{c}{r+c} dA$$

This construction repeated for several points gives us the curve RFGHS. The area between this curve and the axis OGR is

$$\int AF\, dc = \tfrac{1}{2} \int \frac{c}{r+c} dA \tag{21}$$

This integral has a negative value, because the area to the right of OG is greater than the area to the left of OG. Hence the value of Z may be obtained by finding this net area by means of the planimeter and dividing by $-\frac{A}{2}$, the negative half area of the cross section.

Andrew's and Pearson's Theory

A theory which has been approximately verified by experiments, is that developed by E. S. Andrews and Prof. Karl Pearson of the University College of London. The theory as it is here presented is taken from a paper on the subject by Walter Rautenstrauch of Sibley College. It is applied to a hook of the form shown in Fig. 5, and the following assumptions are made.

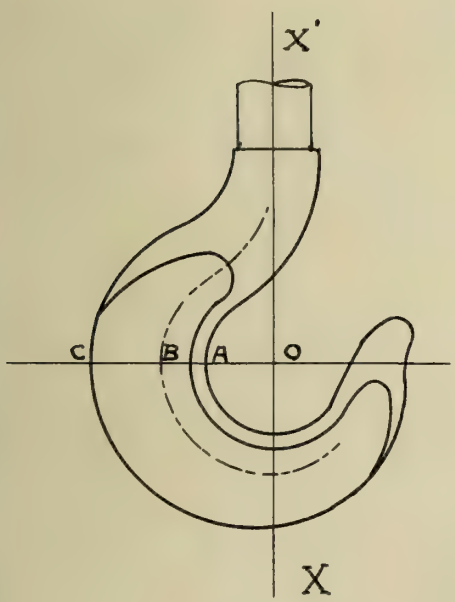


FIG. 5.

The center of curvature of the extrados at C, the intrados at A, and of the neutral axis through B is assumed to be at the point O, which may or may not lie in the load line XX'. The change of curvature near the section is assumed to be very slight and negligible and also that there is no shear on the section ABC, so that the section after strain may be still considered a plane.

In Fig. 6, the full lines represent the position of the section and point O_o before strain, and the broken lines show the deformation due to the strain and the change in the position of the center of curvature from O_o to O'.

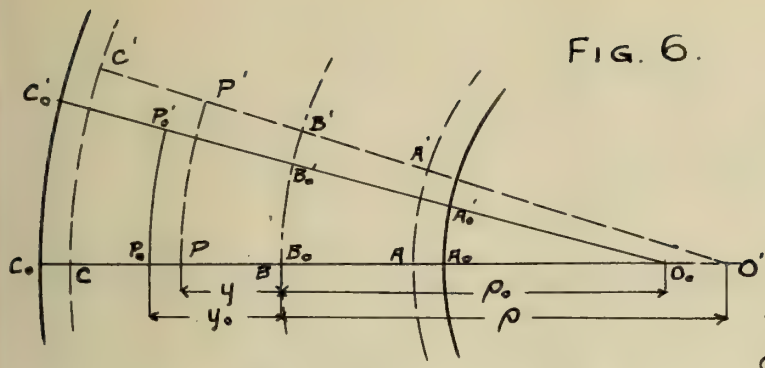


FIG. 6.

Consider a plane passed through O_o, in the immediate neighborhood of the plane $O_o A_o B_o C_o$ and perpendicular to the plane of the paper; and denote the points on this line as indicated. Also consider a point P at a distance y from the gravity axis B of the section, and let S_y represent the unit stretch of the fibres at this point. The tension in this fibre then will be

$$T_y = E S_y \tag{22}$$

where E is the modulus of elasticity.

From Fig. 6.

$$S_y = \frac{PP' - P_o P_o'}{P_o P_o'}$$

$$\text{or} \quad PP' = P_o P_o' (1 + S_y) \tag{23}$$

also $$\frac{PP'}{BB'} = \frac{\rho + y}{\rho}$$

where ρ is the radius of curvature

and $$\frac{P_o P_o'}{B_o B_o'} = \frac{\rho_o + y_o}{\rho_o}$$

Letting S_b equal the stretch at B, we have

$$BB' = B_o B_o' (1 + S_b) \tag{24}$$

By combining above equations we find

$$\left(1 + \frac{y}{\rho}\right)(1 + S_b) = \left(1 + \frac{y_o}{\rho_o}\right)(1 + S_y) \tag{25}$$

From which

$$\frac{1+\frac{y}{\rho}}{1+\frac{y_0}{\rho_0}} = \frac{1+s_y}{1+s_b} = 1 - s_b + s_y \qquad (26)$$

which is approximately true.

$$s_y = s_b + \frac{\frac{y}{\rho} - \frac{y_0}{\rho_0}}{1+\frac{y_0}{\rho_0}}$$

Whence

$$(s_y - s_b)\left(1+\frac{y_0}{\rho_0}\right) = \frac{y}{\rho} - \frac{y_0}{\rho_0} \qquad (27)$$

Differentiating (27),

$$\delta s_y \left(1+\frac{y_0}{\rho_0}\right) + (s_y - s_b)\frac{\delta y_0}{\rho_0} = \frac{\delta y}{\rho} - \frac{\delta y_0}{\rho_0} \qquad (28)$$

The terms δy and δy_0 represent the transverse shortening of the fibres, and the corresponding transverse stress may be obtained as follows.

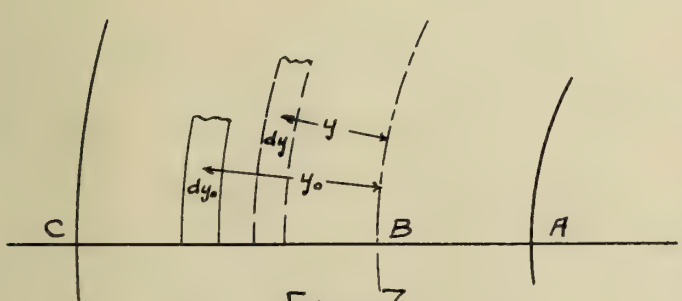


FIG. 7.

In Fig. 7, let the fibre of width δy_0 and distance y_0 from the gravity axis, have a width δy after stress and be at a distance y from the axis.

The transverse shortening we find equal to

$$\frac{\delta y_0 - \delta y}{\delta y_0}$$

This, according to the theory of elasticity, is equal to ηs_y where

η = Poisson's ratio

Whereupon $$\frac{\delta y_o - \delta y}{\delta y_o} = \eta S_y$$

or $$\delta y = \delta y_o (1 - \eta S_y) \qquad (29)$$

Substituting this value of δy in (28)

$$\frac{\delta S_y}{\frac{1}{\rho} - \frac{1}{\rho_o} + \frac{S_b}{\rho_o} - S_y\left(\frac{1}{\rho_o} + \frac{\eta}{\rho}\right)} = \frac{\delta y_o}{1 + \frac{y_o}{\rho_o}} \qquad (30)$$

If we let $S_y\left(\frac{1}{\rho_o} + \frac{\eta}{\rho}\right) = S_y\left(\frac{1+\eta}{\rho_o}\right)$, which is practically true, we get by integrating,

$$-\frac{\rho_o}{1+\eta} \log\left[\frac{1}{\rho} - \frac{1}{\rho_o} + \frac{S_b}{\rho_o} - S_y\left(\frac{1+\eta}{\rho_o}\right)\right] = \rho_o \log\left(1 + \frac{y_o}{\rho_o}\right) + \log C$$

which reduces to

$$\frac{1}{\rho} - \frac{1}{\rho_o} + \frac{S_b}{\rho_o} - S_y\left(\frac{1+\eta}{\rho_o}\right) = \frac{C}{\left(1 + \frac{y_o}{\rho_o}\right)^{1+\eta}} \qquad (31)$$

When $S_y = S_b$, $y_o = 0$

Therefore the value of C is

$$C = \frac{1}{\rho} - \frac{1}{\rho_o} + \frac{\eta S_b}{\rho_o} \qquad (32)$$

Substituting the value of C in (31) and solving for S_y,

$$S_y = \frac{S_b}{1+\eta} + \frac{\rho_o}{1+\eta}\left(\frac{1}{\rho} - \frac{1}{\rho_o}\right) - \frac{\frac{\rho_o}{1+\eta}\left(\frac{1}{\rho} - \frac{1}{\rho_o} - \frac{\eta S_b}{\rho_o}\right)}{\left(1 + \frac{y_o}{\rho_o}\right)^{1+\eta}}$$

or

$$S_y = S_b + \left\{\frac{\rho_o}{1+\eta}\left(\frac{1}{\rho} - \frac{1}{\rho_o}\right) - \frac{\eta S_b}{1+\eta}\right\}\left(1 - \frac{1}{\left(1 + \frac{y_o}{\rho_o}\right)^{1+\eta}}\right)$$

Then the tensile stress T_y in any fibre at a distance of y from the gravity axis is,

$$T_y = ES_b + E\left\{\frac{\rho_o}{1+\eta}\left(\frac{1}{\rho}-\frac{1}{\rho_o}\right) - \frac{\eta S_b}{1+\eta}\right\}\left(1 - \frac{1}{\left(1+\frac{y_o}{\rho_o}\right)^{1+\eta}}\right) \qquad (33)$$

Let W = the load; l = the distance from the load line to the center of gravity; $\Sigma a = A$; that is, the summation of the infinitely small area elements equals the total area.

Then
$$W = \Sigma(T_y a)$$
$$Wl = -\Sigma(T_y a \times y_o)$$

If we let
$$\beta = \frac{\rho_o}{1+\eta}\left(\frac{1}{\rho}-\frac{1}{\rho_o}\right) - \frac{\eta S_b}{1+\eta} \qquad (34)$$

$$A\gamma_1 = \Sigma \frac{a}{\left(1+\frac{y_o}{\rho_o}\right)^{1+\eta}} = \Sigma\left[\left(\frac{\rho_o}{\rho_o+y_o}\right)^{1+\eta} a\right] \qquad (35)$$

$$A\gamma_2 = -\Sigma \frac{a\frac{y_o}{\rho_o}}{\left(1+\frac{y_o}{\rho_o}\right)^{1+\eta}} = -\Sigma\left[\left(\frac{\rho_o}{\rho_o+y_o}\right)^{\eta} a\right] + A\gamma_1 \qquad (36)$$

Then
$$W = EAS_b + E\beta A(1-\gamma_1)$$
$$Wl = -E\beta A\rho_o\gamma_2$$

And
$$S_b = \frac{W}{EA} + \frac{Wl}{EA\rho_o}\,\frac{1-\gamma_1}{\gamma_2}$$

$$\beta = -\frac{Wl}{EA\rho_o\gamma_2}$$

Equating the two values of β,

$$\frac{1}{\rho}-\frac{1}{\rho_o} = \frac{W}{EA}\left\{-\frac{l(1+\eta\gamma_1)}{\gamma_2\rho_o^2} + \frac{\eta}{\rho_o}\right\}$$

$$S_b = \frac{W}{EA}\left\{1 + \frac{l}{\rho_o}\,\frac{1-\gamma_1}{\gamma_2}\right\}$$

Substituting in (33)

$$T_y = \frac{W}{A}\left\{1 + \frac{Z}{\rho_0 \gamma_2}\left(\frac{1}{\left(1+\frac{y_0}{\rho_0}\right)^{1+\eta}} - \gamma_1\right)\right\} \qquad (37)$$

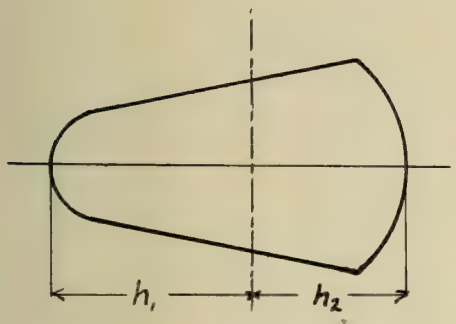


FIG. 8.

When $y_0 = h_1$, then $T_y = -C$ the maximum compressive stress, and when $y_0 = h_2$, then $T_y = T$ the maximum tensile stress. See Fig 8.- The values of T and C are therefore

$$T = \frac{W}{A}\left[\frac{Z}{\rho_0 \gamma_2}\left(\frac{1}{\left(1-\frac{h_2}{\rho_0}\right)^{1+\eta}} - \gamma_1\right) + 1\right] \qquad (38)$$

$$C = \frac{W}{A}\left[\frac{Z}{\rho_0 \gamma_2}\left(\gamma_1 - \frac{1}{\left(1+\frac{h_1}{\rho_0}\right)^{1+\eta}}\right) - 1\right] \qquad (39)$$

These equations may be solved by finding values for γ_1 and γ_2, and a value for η which may be taken as $\frac{1}{4}$.

The following graphical construction has been devised by Mr. Andrews and Prof. Pearson for finding γ_1 and γ_2. See Fig 9.-

Describe any parabola OV with O as a vertex, and OABC as an axis; and also any other parabola OW with O as a vertex and OT as an axis, and let TW be some convenient known length.

Let $CV = a$, $VT = d$, $TW = b$.

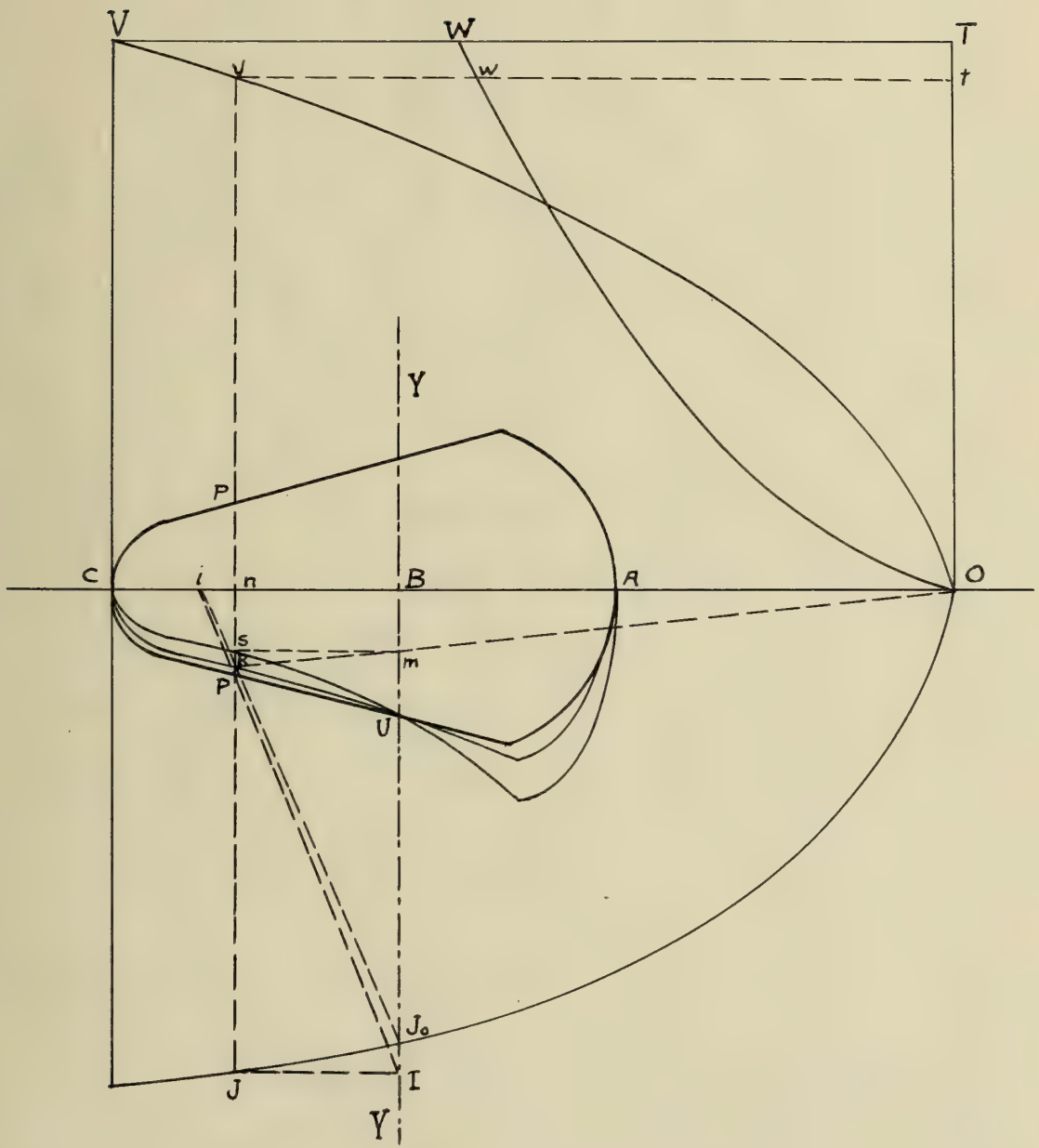


FIG. 9.

Let any line of the hook section $P_n P'$, parallel to YY meet the first parabola in v, and let vwt be parallel to OABC. Then if $tw = Z$

$$\frac{Z^2}{b^2} = \frac{ot}{a} \quad \text{or} \quad \frac{Z^4}{b^4} = \frac{\overline{ot}^2}{a^2} = \frac{\overline{nv}^2}{a^2}$$

But $$\frac{\overline{nv}^2}{a^2} = \frac{on}{oC} = \frac{on}{d}$$

Therefore $$\frac{Z^4}{b^4} = \frac{on}{d}$$

or $$Z = b\left(\frac{on}{d}\right)^{\frac{1}{4}} \tag{40}$$

Let this curve be plotted as shown at OJ, letting $Jn = wt$, etc; let YY meet this curve in J_o and $P_n P'$ in J. Moreover, let IJ meet yy in I; and let IP meet OABC in i and $J_o i$ meet $P_n P'$ in R. Plot the curve CRUR'A, the locus of points R.

Then $$\frac{Rn}{Pn} = \frac{J_o B}{IB} = \frac{J_o B}{Jn} = \frac{\overline{oB}^{\frac{1}{4}}}{\overline{oN}^{\frac{1}{4}}}$$

Therefore $$Rn = Pn\left(\frac{\rho_o}{\rho_o + y_o}\right)^{\frac{1}{4}} \tag{41}$$

Join RO and let it meet yB in m, draw ms parallel to OABC. Plot the curve CSUS'A, the locus of the points s.

Then $$\frac{Sn}{Rn} = \frac{mB}{Rn} = \frac{\rho_o}{\rho_o + y_o}$$

$$Sn = \frac{\rho_o}{\rho_o + y_o} Rn = Pn\left(\frac{\rho_o}{\rho_o + y_o}\right)^{\frac{5}{4}} \tag{42}$$

We find then that

$$\gamma_1 = \frac{\text{area } CSUS'A}{\frac{1}{2}\text{ area of section}} \qquad (43)$$

$$\gamma_2 = \frac{\text{area } CSUS'A - \text{area } CRUR'A}{\frac{1}{2}\text{ area of section}} \qquad (44)$$

Theory of Slocum and Hancock.

In the development of this theory consider a curved piece of circular cross section, and center of curvature at O, as in Fig 10.

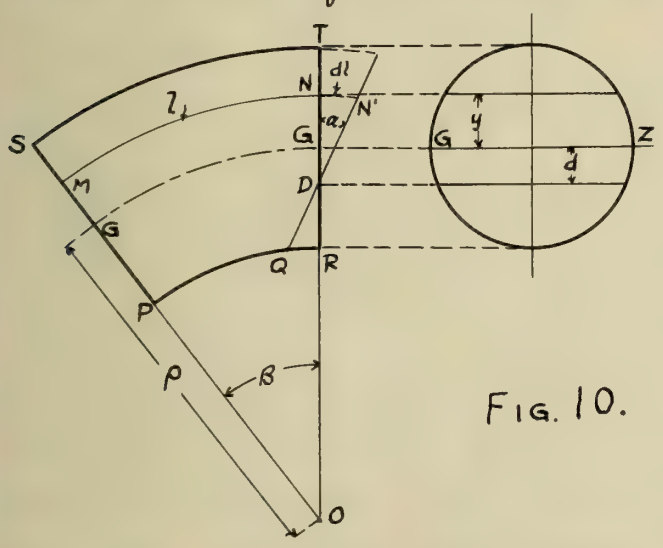


FIG. 10.

Hook's Law and Bernoulli's assumption are taken as a starting point; that is, that the stress is proportional to the deformation, and that any plane section remains plane during the deformation. Since the fibres on the convex side are longer than those on the concave side, it will take less stress to deform them an equal amount, and therefore the neutral axis does not lie in the gravity axis, but is at a distance d from it, as shown in the figure. The deformation of the fibres above the axis is therefore greater than that below.

Let the length of any fibre MN be l, and its distance from the gravity axis y, and its deformation dl. Let ρ = the radius of curvature, OG of the piece, β the angle between two plane sections, and α the angle of deformation of a plane section.

Then $$l = \beta \cdot MO = (OG + GN)\beta = (\rho + y)\beta \qquad (45)$$

$$dl = NN' = \alpha \cdot ND = (y + d)\alpha \qquad (46)$$

From Hook's Law

$$\frac{dl}{l} = \frac{p}{E}$$

where p = the unit stress, and E = the modulus of elasticity.

Then $$p = \frac{E\,dl}{l} = \frac{E(y+d)\alpha}{(\rho+y)\beta}$$

Let $\frac{\alpha}{\beta} = k$, a constant.

$$p = Ek\,\frac{y+d}{\rho+y} \qquad (47)$$

Assuming a pure bending strain, the shear on the section is zero, and the normal stresses form a couple. Therefore the algebraic sum of these stresses is zero, or

$$\int p\,dA = 0$$

where dA is an infinitely small area element.

Substituting the value of p in (47)

$$kE \int \frac{y+d}{\rho+y}\,dA = 0$$

Since k and E are constants and not zero,

the integral must be zero. Separating the integral into parts,

$$\int \frac{y\,dA}{y+\rho} + d\int \frac{dA}{y+\rho} = 0$$

or
$$d = -\frac{\int \frac{y\,dA}{y+\rho}}{\int \frac{dA}{y+\rho}}, \qquad (48)$$

which shows that the neutral axis is below the gravity axis. Let M denote the external bending moment on a given section of area A, and y its distance from the gravity axis.

Then $\int (y+d)\,p\,dA = M$

Substituting the value of p from (47),

$$Ek\int \frac{(y+d)^2}{y+\rho}\,dA = M$$

$$k = \frac{M}{E\int \frac{(y+d)^2}{y+\rho}\,dA}.$$

and
$$p = Ek\,\frac{y+d}{y+\rho} = \frac{M(y+d)}{(y+\rho)\int \frac{(y+d)^2}{y+\rho}\,dA} \qquad (49)$$

Therefore by substituting the value of d from (48), in (49) the value of the unit stress p may be found.

For the application of this formula, a geometrical transformation due to Résal

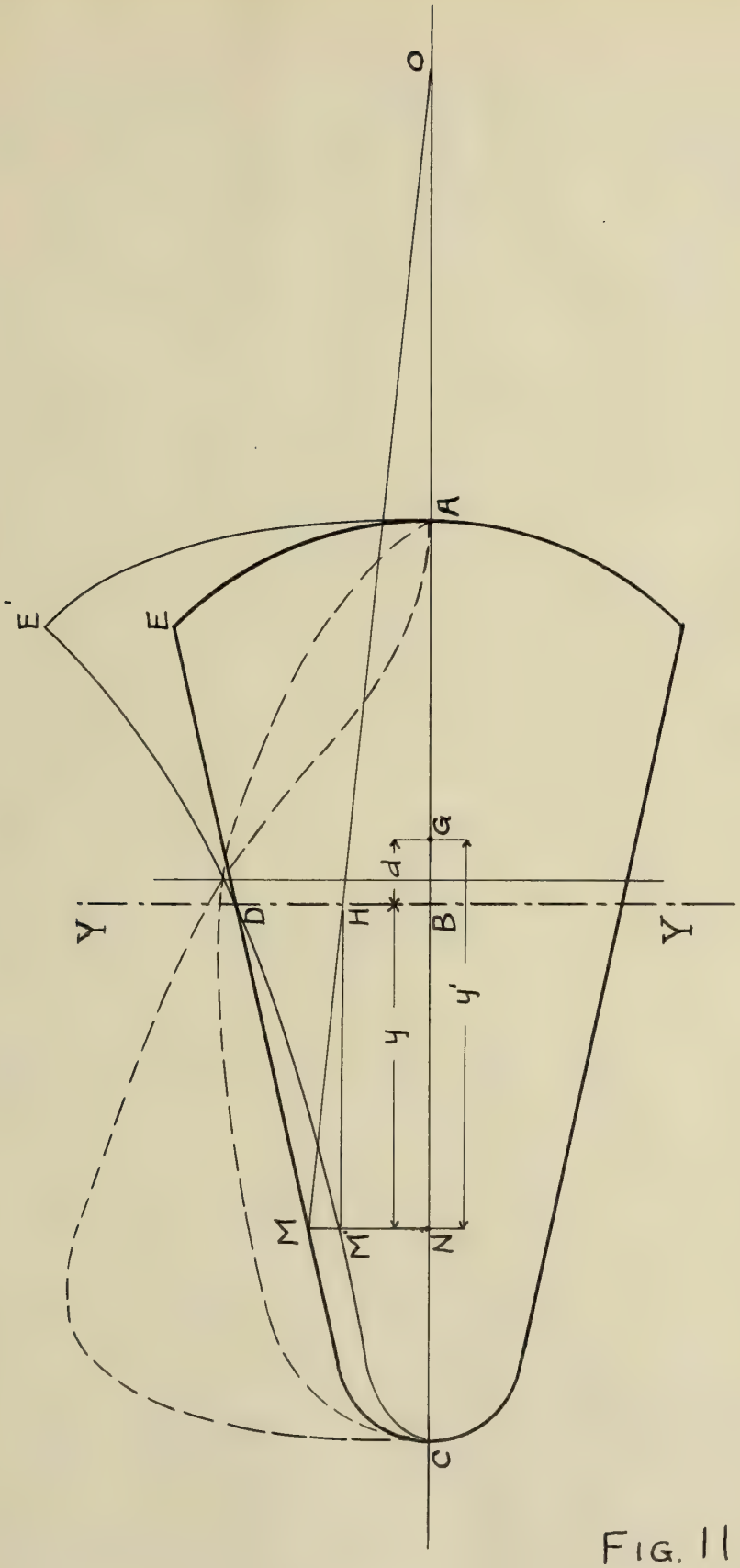


FIG. 11.

greatly simplifies the operation. The first step of the construction is to transform the section as shown in Fig 11, at ABCM'DE'.

Take any point M on the boundary of the original section and draw OM, intersecting YY the gravity axis in H. Draw HM' parallel to OABC, intersecting NM in M'. Draw the curve CM'DE', the locus of the points M'.

The distance of the center of gravity of the transformed section from that of the original section is the value of d (equation 48); and the moment of inertia I' of this transformed section is equal to the value of the integral in equation (49).

That is $$I' = \rho \int \frac{(y+d)^2}{(y+\rho)} dA$$

This may be shown to be true as follows:—

Let NM' be denoted by z',

Then $$NM' = z' = BH = MN \frac{OB}{ON} = z \frac{\rho}{\rho+y} \qquad (50)$$

The distance d' of the center of gravity of the transformed section from that of the original section is

$$d' = BG = -\frac{\int z'y\,dy}{\int z'dy} = -\frac{\int zy \frac{\rho}{\rho+y}\,dy}{\int z \frac{\rho}{\rho+y}\,dy}$$

Dividing out the constant p and replacing $z\,dy$ by dA, the expression becomes

$$d' = -\frac{\int \frac{y\,dA}{y+p}}{\int \frac{dA}{y+p}}, \tag{51}$$

which is equal to d in equation (48)

Let dA' be an element of area of the transformed section, and y' its distance from the center of gravity of the new section. Then from the expression for the moment of inertia of an area,

$$I' = \int y'^2 dA'$$

$$\text{Since } dA' = z'dy';\ y' = y+d\ ;\ z' = z\frac{p}{p+y}\ ;\ dy' = dy \tag{52}$$

$$I' = \int (y+d)^2 \left(z\frac{p}{p+y}\right) dy$$

$$\text{or} \quad I' = p\int \frac{(y+d)^2}{y+p} dA\ , \text{ since } z\,dy = dA. \tag{53}$$

Formula (49) may now be written

$$P = \frac{Mp}{I'}\left(\frac{y+d}{y+p}\right) \tag{54}$$

To this value of p must be added the uniform stress $\frac{Load}{A}$ for tension, and subtracted for compression.

$$T = \frac{P}{A} + \frac{Mp}{I'}\left(\frac{y+d}{y+p}\right) \tag{55}$$

$$C = \frac{P}{A} - \frac{Mp}{I'}\left(\frac{y-d}{y+p}\right) \tag{56}$$

Sample Calculations

Calculations for the stresses in the horizontal section of the 15 Ton Ide and Son hook according to the four theories are as follows :-

(a) Unwin

$$T = \frac{P}{A} + \frac{P p c_t}{I} = \frac{30000}{18.344} + \frac{30000 \times 7.698 \times 3.323}{67.56} = 13000^{\#}$$

$$C = \frac{P}{A} - \frac{P p c_c}{I} = \frac{30000}{18.344} - \frac{30000 \times 7.698 \times 4.052}{67.56} = -12215^{\#}$$

(b) Bach

$$T = \frac{P}{AZ}\ \frac{c_t}{r - c_t} = \frac{30000}{18.344 \times .0764}\ \frac{3.323}{4.375} = 16255^{\#}$$

$$C = \frac{P}{AZ}\ \frac{c_c}{r + c_c} = \frac{30000}{18.344 \times .0764}\ \frac{4.052}{11.75} = -7380^{\#}$$

(c) Rautenstrauch

$$T = \frac{W}{A}\left[\frac{Z}{p_o \gamma_2}\left\{\frac{1}{\left(1 - \frac{h_2}{p_o}\right)^{\frac{5}{4}}} - \gamma_1\right\} + 1\right] = \frac{30000}{18.344}\left[\frac{7.698}{7.698 \times 0.0938}\left\{\frac{1}{\left(1 - \frac{3.323}{7.698}\right)^{\frac{5}{4}}} - 1.11\right\} + 1\right] =$$

$$17670^{\#}$$

$$C = \frac{W}{A}\left[\frac{Z}{p_o \gamma_2}\left\{\frac{1}{\left(1 + \frac{h_1}{p_o}\right)^{5/4}} + \gamma_1\right\} - 1\right] = \frac{30000}{18344}\left[\frac{7.698}{7.698 \times .0938}\left\{1.11 - \frac{1}{\left(1 + \frac{4.052}{7.698}\right)^{5/4}}\right\} - 1\right] =$$

$$-7260^{\#}$$

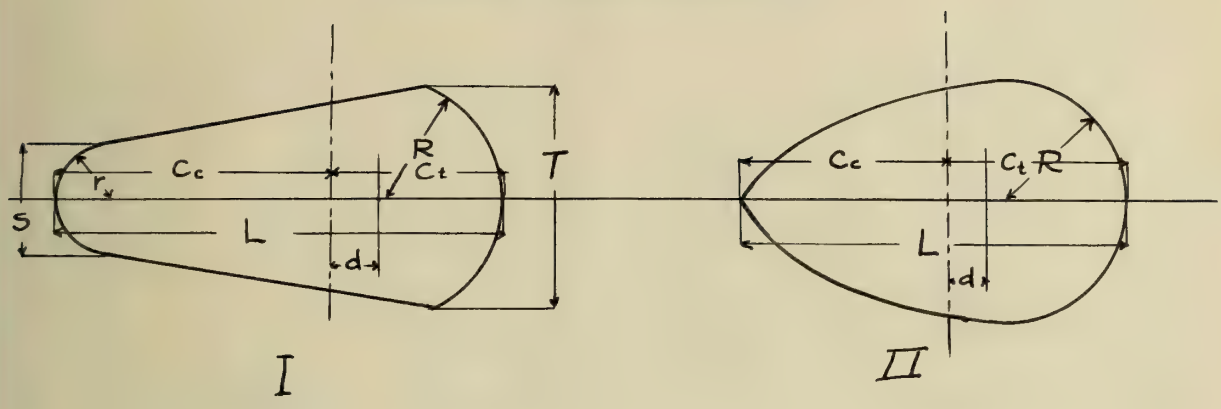
(d) Slocum and Hancock.

$$T = \frac{P}{A} + \frac{M_p}{I'}\,\frac{y+d}{p-y} = \frac{30000}{18.344} + \frac{30000 \times 7.145 \times 7.698}{91.2}\,\frac{2.77}{4.375} = 13055^{\#}$$

$$C = \frac{P}{A} - \frac{M_p}{I'}\,\frac{y-d}{p+y} = \frac{30000}{18.344} - \frac{30000 \times 7.145 \times 7.698}{91.2}\,\frac{4.605}{11.75} = -5445^{\#}$$

Note:- The quantity d is a negative one.

HOOK DIMENSIONS.



1ST HOOK. 12 1/2 TONS.

SECTION		- DIMENSIONS - INCHES. -					
Location.	Shape	L	R	T	r	C_t	C_c
45° Ab. H	I	4 1/4	2	2 7/8	3/4	1.87	2.38
H	I	5 1/8	2	2 7/8	1/2	2.11	3.115
45° Bel. H	I	4 3/4	2	2 7/8	1/2	2.02	2.73

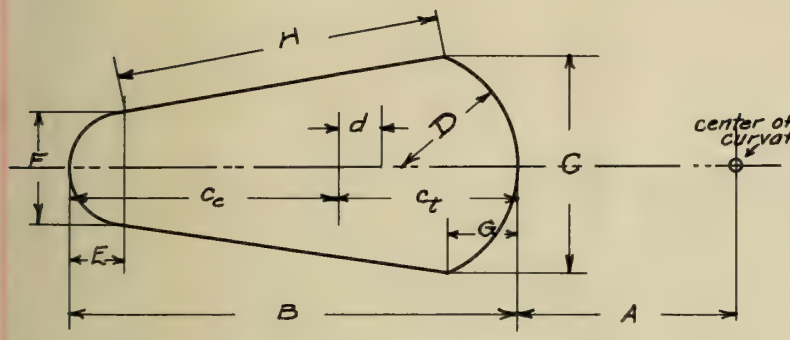
2ND HOOK. 5 TONS.

SECTION.		- DIMENSIONS. - INCHES. -					
Location.	Shape	L	R	T	r	C_t	C_c
45° Ab. H	I	2 3/8	1"	2 1/16	13/16	1.16	1.215
H	I	2 13/16	15/16	1 3/4	3/8	1.21	1.6
45° Bel. H	I	2 3/4	15/16	1 3/4	3/8	1.2	1.55

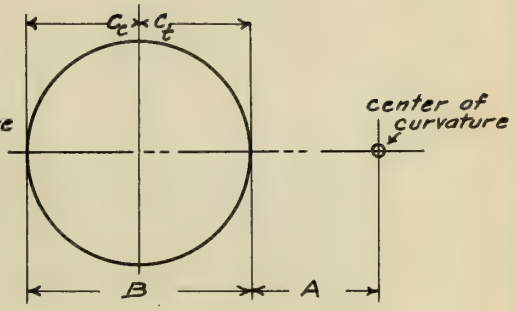
3RD HOOK. 5 TONS.

SECTION		- DIMENSIONS - INCHES. -					
Location	Shape	L	R	T	r	C_t	C_c
45° Ab. H.	II	1 15/16	3/4"	—	—	.895	1.045
H	II	2 3/8	3/4"	—	—	1.06	1.315
45° Bel. H	II	2 3/16	3/4"	—	—	.967	1.22

-HOOK DIMENSIONS-



TRAPEZOIDAL SECTION.-



CIRCULAR SECTION.-

HOOK NO. 1- IDE & SON - 15 TONS-

SECTION		DIMENSIONS - INCHES-										
Location	Shape	A	B	C	D	E	F	G	H	d	C_t	C_c
45° Above H.	TRAP.	5.000	6.312	4.188	2.375	0.875	1.750	1.250	------	0.442	2.885	3.428
Horizontal	"	4.375	7.375	3.562	2.000	0.875	1.750	1.094	-----	0.553	3.323	4.052
45° Below H.	"	4.375	7.062	3.562	2.000	0.875	1.750	1.094	----	0.500	3.157	3.905

HOOK NO 2- WHITING FOUNDRY CO- 10 TONS-

SECTION		DIMENSIONS - INCHES-										
Location	Shape	A	B	C	D	E	F	G	H	d	C_t	C_c
45° Above H.	CIRC.	2.625	3.625	----	----	---	----	----	----	0.20	1.812	1.812
Horizontal	TRAP.	2.625	4.25	3.00	----	0.25	1.75	----	3.50	0.315	1.98	2.27
45° Below H.	"	2.625	4.125	2.625	----	0.25	1.25	----	3.375	0.25	1.865	2.26

HOOK NO. 3- WALKER MFG. CO.- 10 TONS-

SECTION		DIMENSIONS - INCHES-										
Location	Shape	A	B	C	D	E	F	G	H	d	C_t	C_c
45° Above H.	CIRC.	3.50	4.25	----	----	-----	----	----	-----	0.215	2.125	2.125
Horizontal	"	3.50	4.50	----	----	----	-----	----	-----	0.22	2.25	2.25
45° Below H.	"	3.50	4.50	----	----	-----	-----	-----	-----	0.22	2.25	2.25

Hook Constants.

1st Hook. 12 ½ Tons.

Section	Area	I	Bach	Rautenstrauch		Slocum	
			Z	γ_1	γ_2	I'	d
45°Ab.H	8.6	11.80	.060	1.08	.0805	9.28	.22
H	9.6	18.24	.066	1.11	.110	18.40	.36
45°Bel.H	9.2	15.60	.0625	1.075	.075	12.40	.22

2nd Hook. 5 Tons.

Section	Area	I	Bach	Rautenstrauch		Slocum	
			Z	γ_1	γ_2	I'	d
45°Ab.H.	3.64	1.28	.0495	1.06	.044	1.44	.11
H	3.40	1.55	.0410	1.06	.065	1.64	.13
45°Bel.H	3.40	1.55	.0416	1.07	.065	1.50	.13

3rd Hook. 5 Tons.

Section	Area	I	Bach	Rautenstrauch		Slocum	
			Z	γ_1	γ_2	I'	d
45°Ab.H.	2.10	.9	.050	1.06	.0475	.962	.095
H	2.50	1.8	.060	1.11	.090	1.60	.135
45°Bel.H.	2.325	1.4	.0516	1.075	.0645	1.30	.097

Hook Constants-

Hook No. 1- Ide & Son - 15 Tons-

Section	Area	Moment of Inertia	Bach	Rautenstrauch		Slocum
			Z	γ_1	γ_2	I'
45° Above Horiz.	16.404	37.30	0.0695	1.172	0.0854	45.24
Horizontal	18.344	67.56	0.0764	1.110	0.0938	91.20
45° Below Horiz.	17.514	61.04	0.0696	1.102	0.0857	60.10

Hook No 2 - Whiting Foundry Co- 10 Tons-

Section	Area	Moment of Inertia	Bach	Rautenstrauch.		Slocum
			Z	γ_1	γ_2	I'
45° Above Horiz	10.321	8.46	0.0454	1.064	0.0542	9.05
Horizontal	9.62	12.56	0.0718	1.096	0.0874	12.30
45° Below Horiz	7.66	9.44	0.0757	1.092	0.0927	8.90

Hook No 3.- Walker Mfg. Co- 10 Tons-

Section	Area	Moment of Inertia	Bach	Rautenstrauch		Slocum
			Z	γ_1	γ_2	I'
45° Above Horiz.	14.186	16.0	0.0385	1.056	0.0557	16.84
Horizontal	15.904	20.1	0.0414	1.069	0.0566	16.60
45° Below Horiz.	15.904	20.1	0.0414	1.069	0.0566	16.60

—TABLE OF MAXIMUM STRESSES.—

1ST HOOK. 12½ TONS.

SECTION	UNWIN		BACH		RAUTENSTRAUCH		SLOCUM.	
	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.
45°Ab. H.	11100	8600	23200	12000	20200	10000	21200	10100
H.	16000	10800	35000	18100	27000	11500	22600	12400
45°Bel. H	11500	8750	21600	11000	22600	10100	22300	10840

2ND HOOK. 5 TONS.

SECTION	UNWIN		BACH		RAUTENSTRAUCH		SLOCUM	
	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.
45°Ab. H.	18300	18900	24000	11300	21950	9240	27360	14200
H	26940	28000	46000	24200	39600	18300	36240	20560
45°Bel. H.	19260	19300	32000	16800	29560	13300	30300	15600

3RD HOOK. 5 TONS.

SECTION	UNWIN		BACH		RAUTENSTRAUCH		SLOCUM.	
	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.
45°Ab. H.	13400	10240	40000	21000	32000	14000	26400	11210
H.	19000	14700	47000	22800	41000	19620	29200	16100
45°Bel. H.	16230	12500	38000	18800	31200	15100	26400	11390

-.TABLE OF MAXIMUM STRESSES.-

HOOK NO1- IDE & SON - 15 TONS.-

Section	UNWIN		BACH		RAUTENSTRAUCH		SLOCUM	
	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.
45° Above Horiz.	13325	13000	12475	5155	10210	5700	13695	7425
Horizontal	13000	12215	16255	7380	17670	7260	13055	5445
45° Below Horiz.	9555	8905	15060	5855	14050	5475	12645	5920

HOOK NO.2- WAITING FOUNDRY CO.- 10 TONS-

Section	UNWIN		BACH		RAUTENSTRAUCH		SLOCUM.	
	TENS.	COMP	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.
45° Above Horiz.	14810	12070	23725	8740	19820	8475	19440	8110
Horizontal	16290	14590	21850	9560	24080	9600	22480	10020
45° Below Horiz.	14425	13380	21000	8160	19860	7170	20475	9400

HOOK NO.3- WALKER MFG. CO.- 10 TONS-

Section	UNWIN		BACH		RAUTENSTRAUCH		SLOCUM	
	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.	TENS.	COMP.
45° Above Horiz.	11575	9575	17780	7125	14920	5510	14960	6625
Horizontal	14130	11620	19550	8550	18825	7800	23455	10565
45° Below Horiz.	10000	8225	15600	6050	13700	5150	16580	7470

Conclusion -

The theory of Unwin, as stated before, disregards the fact that the crane hook is a curved bar, and we presume that the stresses obtained by using his formula are far from correct. A glance at the table of maximum stresses shows that Unwin's values differ by from 25% to 40% from those given by the other three theories, Bach, Andrew and Pearsons, and Slocum and Hancock. Moreover the latter three theories give stresses which agree fairly well, the average difference being approximately 10%.

That these three theories are better than Unwin's, is seen from some experiments made by Mr. Andrews and Prof. Pearson. By actual test, a railway car coupling hook was found to have an elastic limit of 21000 pounds per square inch. A computation of the stress according to Unwin's formula gave a stress of 12000 pounds per square inch, while Andrews and Pearson's formula gave 23000 pounds per square inch. Another hook with a T cross section was tested and the actual stress at the elastic limit was found to be 30000 pounds

per square inch. The respective values of the calculated stresses were 18700 pounds per square inch and 29300 pounds per square inch.

From the experiments made by Mr. Andrews and Prof. Pearson, it is apparent that the old theory of Unwin errs a great deal, while the new theory is on the safe side. The tables of maximum stresses seem to show that Bach's Theory, and Andrews's and Pearson's Theory give values that agree best, while Slocum and Hancock's Theory agrees quite well with either one. It is therefore safe to assume that the design of a hook, according to either one of the three methods, gives a sufficiently accurate analysis of the stresses to be within the bounds of safety.